

# Art Project Isaac Newton

We started this project from our desire to have our field of study represented on the Campus in an art work. We have chosen the life and work of Sir Isaac Newton, the founder of the modern technological and mathematical theories as a starting point of this project

We would like to see an art work that is both artistically exciting and technically sound. To achieve that, we would like to combine the strength of both technical and artistic students and we offer our knowledge and expertise to the students of the DAI in the development of the proposals. We hope the students of both Twente University and the DAI can work together, help each other and learn from the experience.

## Isaac Newton

Sir Isaac Newton was an English physicist, mathematician, astronomer, natural philosopher and alchemist, generally regarded as one of the greatest figures in the history of science. His famous three laws of motion are the groundwork for classical mechanics. He was the first to show that the motions of objects on earth and of objects in space (like planets) are governed by the same set of natural laws. In optics, he invented the reflecting telescope and developed a theory of colour based on the observation that a prism decomposes white light into a visible spectrum. He also formulated a law of cooling, studied the speed of sound and proposed a theory of the origin of stars. In mathematics Newton played a significant role in the development of calculus. A lot more information about Isaac Newton can be found on the internet. As an example we have added the wikipedia page about Isaac Newton and some pages about his laws of motion. On the wikipedia page are a lot of links which can be useful. Some of them, which are in *Italic* can especially be useful, because they explain the experiments and life of Isaac Newton in a very clear way.

The artwork is to be placed in front, on top, or maybe integrated with the 'Horst' building on the campus. We ask you to come up with a proposal in December, you can always get in touch with us if you have any questions on technical details. Then in January we would like you to present your proposals in an exhibition in the Horst building. A combined jury of students, employees and the art commission will choose a proposal and we are going to realize the work, after getting necessary permissions and enough funding.

There are more artists who have integrated technology and art. Here are some examples on the internet:

[www.boschsimons.com](http://www.boschsimons.com)

[www.bramvreven.com](http://www.bramvreven.com)

[http://nl.wikipedia.org/wiki/Wim T. Schippers](http://nl.wikipedia.org/wiki/Wim_T._Schipper)

<http://www.cultuurwijs.nl/cultuurwijs.nl/cultuurwijs.nl/i000980.html>

[www.panamarenko.org](http://www.panamarenko.org)

[www.tinguely-jean.com](http://www.tinguely-jean.com) google tinguely images

You can always contact us if you want to:

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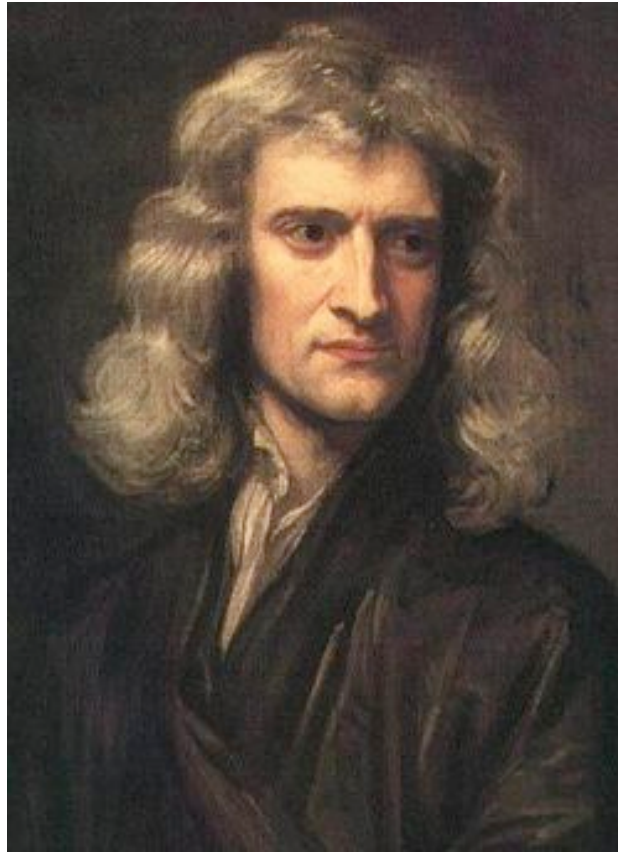
Remco : 06-22951190

[r.n.brantjes@student.utwente.nl](mailto:r.n.brantjes@student.utwente.nl)

Gerwin : 06-43063702

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## Sir Isaac Newton



*Isaac Newton at 46 in  
Godfrey Kneller's 1689 portrait*

Born	4 January 1643(1643-01-04) [OS: 25 December 1642][1] Woolsthorpe-by-Colsterworth, Lincolnshire, England
Died	31 March 1728 (aged 85) [OS: 20 March 1727][1] Kensington, London, England
Residence	England
Nationality	English
Field	Theologian, Physicist, mathematician, astronomer, natural philosopher, and alchemist
Institutions	University of Cambridge Royal Society
Alma mater	Trinity College, Cambridge
Known for	Newtonian mechanics Universal gravitation Infinitesimal calculus Classical optics

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# 1 Biography

## 1.1 Early years



*Newton in a 1702 portrait by Godfrey Kneller.*

Isaac Newton was born on January 4, 1643 [OS: December 25, 1642]<sup>[1]</sup> at Woolsthorpe Manor in Woolsthorpe-by-Colsterworth, a hamlet in the county of Lincolnshire. At the time of Newton's birth, England had not adopted the latest papal calendar and therefore his date of birth was recorded as Christmas Day, December 25, 1642. Newton was born three months after his father, also called Isaac, died. Born prematurely, he was a small child; his mother Hannah Ayscough reportedly said that he could have fit inside a quart mug. When Newton was three, his mother remarried and went to live with her new husband, the Reverend Barnabus Smith, leaving her son in the care of his maternal grandmother, Margery Ayscough. The young Isaac disliked his step-father and held some enmity towards his mother for marrying him, as revealed by this entry in a list of sins committed up to the age of 19: *Threatening my father and mother Smith to burn them and the house over them.*<sup>[3]</sup>

Newton is believed by some researchers to have suffered from Asperger's Syndrome, a form of autism.<sup>[4][5]</sup> Indeed it is believed that like other historical geniuses Asperger's may have been the very cause of Newton's intellect.

According to E.T. Bell and H. Eves: *Newton began his schooling in the village schools and was later sent to The King's School, Grantham, where he became the top student in the school. At King's, he lodged with the local apothecary, William Clarke and eventually became engaged to the apothecary's stepdaughter, Anne Storer, before he went off to the University of Cambridge at the age of 19. As Newton became engrossed in his studies, the romance cooled and Miss Storer married someone else. It is said he kept a warm memory of this love, but Newton had no other recorded "sweet-hearts" and never married.*<sup>[6]</sup>

He is suspected to have been a virgin throughout his life.<sup>[7]</sup> However, Bell and Eves' sources for this claim, William Stukeley and Mrs. Vincent (the former Miss Storer — actually named Katherine, not Anne), merely say that Newton entertained "a passion" for Storer while he lodged at the Clarke house.

From the age of about twelve until he was seventeen, Newton was educated at The King's School, Grantham (where his signature can still be seen upon a library window sill). He was removed from school, and by October 1659, he was to be found at Woolsthorpe-by-Colsterworth, where his mother, widowed by now for a second time, attempted to make a farmer of him. He was, by later reports of his contemporaries, thoroughly unhappy with the work. It appears to have been Henry

Stokes, master at the King's School, who persuaded his mother to send him back to school so that he might complete his education. This he did at the age of eighteen, achieving an admirable final report.

In June 1661, he was admitted to Trinity College, Cambridge. At that time, the college's teachings were based on those of Aristotle, but Newton preferred to read the more advanced ideas of modern philosophers such as Descartes and astronomers such as Galileo, Copernicus and Kepler. In 1665, he discovered the generalized binomial theorem and began to develop a mathematical theory that would later become calculus. Soon after Newton had obtained his degree in April of 1665, the University closed down as a precaution against the Great Plague. For the next 2 years, Newton worked at his home in Woolsthorpe on calculus, optics and the law of gravitation.

## 1.2 Middle years



Isaac Newton (Bolton, Sarah K. *Famous Men of Science*. NY: Thomas Y. Crowell & Co., 1889)

### 1.2.1 Mathematics

Most modern historians believe that Newton and Leibniz had developed calculus independently, using their own unique notations. According to Newton's inner circle, Newton had worked out his method years before Leibniz, yet he published almost nothing about it until 1693, and did not give a full account until 1704. Meanwhile, Leibniz began publishing a full account of his methods in 1684. Moreover, Leibniz's notation and "differential Method" were universally adopted on the Continent, and after 1820 or so, in the British Empire. Whereas Leibniz's notebooks show the advancement of the ideas from early stages until maturity, there is only the end product in Newton's known notes. Newton claimed that he had been reluctant to publish his calculus because he feared being mocked for it. Newton had a very close relationship with Swiss mathematician Nicolas Fatio de Duillier, who from the beginning was impressed by Newton's gravitational theory. In 1691 Duillier planned to prepare a new version of Newton's *Philosophiae Naturalis Principia Mathematica*, but never finished it. Some of Newton's biographers have suggested that the relationship may have been romantic.<sup>[8]</sup> However, in 1694 the relationship between the two men cooled down. At the time, Duillier had also exchanged several letters with Leibniz.

Starting in 1699, other members of the Royal Society (of which Newton was a member) accused Leibniz of plagiarism, and the dispute broke out in full force in 1711. Newton's Royal Society proclaimed in a study that it was Newton who was the true discoverer and labeled Leibniz a fraud. This study was cast into doubt when it was later found that Newton himself wrote the study's

concluding remarks on Leibniz. Thus began the bitter Newton v. Leibniz calculus controversy, which marred the lives of both Newton and Leibniz until the latter's death in 1716.

Newton is generally credited with the generalized binomial theorem, valid for any exponent. He discovered Newton's identities, Newton's method, classified cubic plane curves (polynomials of degree three in two variables), made substantial contributions to the theory of finite differences, and was the first to use fractional indices and to employ coordinate geometry to derive solutions to Diophantine equations. He approximated partial sums of the harmonic series by logarithms (a precursor to Euler's summation formula), and was the first to use power series with confidence and to revert power series. He also discovered a new formula for calculating pi.

He was elected Lucasian Professor of Mathematics in 1669. In that day, any fellow of Cambridge or Oxford had to be an ordained Anglican priest. However, the terms of the Lucasian professorship required that the holder *not* be active in the church (presumably so as to have more time for science). Newton argued that this should exempt him from the ordination requirement, and Charles II, whose permission was needed, accepted this argument. Thus a conflict between Newton's religious views and Anglican orthodoxy was averted.

### 1.2.2 Optics

From 1670 to 1672, Newton lectured on optics. During this period he investigated the refraction of light, demonstrating that a prism could decompose white light into a spectrum of colors, and that a lens and a second prism could recombine the multicolored spectrum into white light.



*A replica of Newton's 6-inch reflecting telescope of 1672 for the Royal Society.*

He also showed that the colored light does not change its properties, by separating out a colored beam and shining it on various objects. Newton noted that regardless of whether it was reflected or scattered or transmitted, it stayed the same color. Thus the colors we observe are the result of how objects interact with the incident *already-colored* light, **not** the result of objects *generating* the color. For more details, see Newton's theory of color.

From this work he concluded that any refracting telescope would suffer from the dispersion of light into colors, and invented a reflecting telescope (today known as a Newtonian telescope) to bypass that problem. By grinding his own mirrors, using Newton's rings to judge the quality of the optics for his telescopes, he was able to produce a superior instrument to the refracting telescope, due primarily to the wider diameter of the mirror. In 1671 the Royal Society asked for a demonstration of his reflecting telescope. Their interest encouraged him to publish his notes *On Color*, which he later expanded into his *Opticks*. When Robert Hooke criticised some of Newton's ideas, Newton

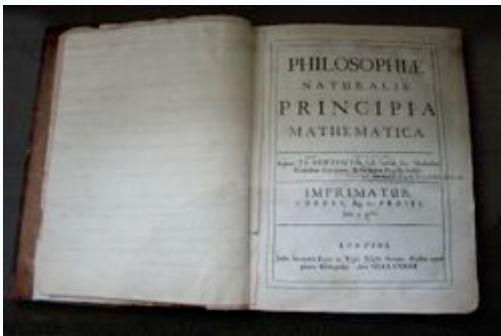
was so offended that he withdrew from public debate. The two men remained enemies until Hooke's death.

Newton argued that light is composed of particles or *corpuscles* and were refracted by accelerating toward the denser medium, but he had to associate them with waves to explain the diffraction of light (*Opticks* Bk. II, Props. XII-L). Later physicists instead favoured a purely wavelike explanation of light to account for diffraction. Today's quantum mechanics, photons and the idea of wave-particle duality bear only a minor resemblance to Newton's understanding of light.

In his *Hypothesis of Light* of 1675, Newton posited the existence of the ether to transmit forces between particles. The contact with the theosophist Henry More, revived his interest in alchemy. He replaced the ether with occult forces based on Hermetic ideas of attraction and repulsion between particles. John Maynard Keynes, who acquired many of Newton's writings on alchemy, stated that "Newton was not the first of the age of reason: he was the last of the magicians."<sup>[9]</sup> Newton's interest in alchemy cannot be isolated from his contributions to science.<sup>[10]</sup> (This was at a time when there was no clear distinction between alchemy and science.) Had he not relied on the occult idea of action at a distance, across a vacuum, he might not have developed his theory of gravity. (See also Isaac Newton's occult studies.)

In 1704 Newton wrote *Opticks*, in which he expounded his corpuscular theory of light. He considered light to be made up of extremely subtle corpuscles, that ordinary matter was made of grosser corpuscles and speculated that through a kind of alchemical transmutation "Are not gross Bodies and Light convertible into one another, ...and may not Bodies receive much of their Activity from the Particles of Light which enter their Composition?"<sup>[11]</sup> Newton also constructed a primitive form of a frictional electrostatic generator, using a glass globe (*Optics*, 8th Query).

### 1.2.3 Mechanics and gravitation



*Newton's own copy of his Principia, with hand-written corrections for the second edition.*  
*Further information: The writing of Principia Mathematica*

In 1677, Newton returned to his work on mechanics, i.e., gravitation and its effect on the orbits of planets, with reference to Kepler's laws of planetary motion, and consulting with Hooke and Flamsteed on the subject. He published his results in *De Motu Corporum* (1684). This contained the beginnings of the laws of motion that would inform the *Principia*.

The *Philosophiæ Naturalis Principia Mathematica* (now known as the *Principia*) was published on 5 July 1687 with encouragement and financial help from Edmond Halley. In this work Newton stated the three universal laws of motion that were not to be improved upon for more than two hundred years. He used the Latin word *gravitas* (weight) for the force that would become known as gravity, and defined the law of universal gravitation. In the same work he presented the first analytical determination, based on Boyle's law, of the speed of sound in air.

With the *Principia*, Newton became internationally recognised. He acquired a circle of admirers, including the Swiss-born mathematician Nicolas Fatio de Duillier, with whom he formed an intense relationship that lasted until 1693. The end of this friendship led Newton to a nervous breakdown.

### 1.3 Later life



*Isaac Newton in 1712. Portrait by Sir James Thornhill.*

In the 1690s Newton wrote a number of religious tracts dealing with the literal interpretation of the Bible. Henry More's belief in the universe and rejection of Cartesian dualism may have influenced Newton's religious ideas. A manuscript he sent to John Locke in which he disputed the existence of the Trinity was never published. Later works — *The Chronology of Ancient Kingdoms Amended* (1728) and *Observations Upon the Prophecies of Daniel and the Apocalypse of St. John* (1733) — were published after his death. He also devoted a great deal of time to alchemy (see above).

Newton was also a member of the Parliament of England from 1689 to 1690 and in 1701, but his only recorded comments were to complain about a cold draft in the chamber and request that the window be closed.

Newton moved to London to take up the post of warden of the Royal Mint in 1696, a position that he had obtained through the patronage of Charles Montagu, 1st Earl of Halifax, then Chancellor of the Exchequer. He took charge of England's great recoinage, somewhat treading on the toes of Master Lucas (and securing the job of deputy comptroller of the temporary Chester branch for Edmond Halley). Newton became perhaps the best-known Master of the Mint upon Lucas' death in 1699, a position Newton held until his death. These appointments were intended as sinecures, but Newton took them seriously, retiring from his Cambridge duties in 1701, and exercising his power to reform the currency and punish clippers and counterfeiters. As Master of the Mint in 1717 Newton unofficially moved the Pound Sterling from the silver standard to the gold standard by creating a relationship between gold coins and the silver penny in the "Law of Queen Anne"; these were all great reforms at the time, adding considerably to the wealth and stability of England. It was his work at the Mint, rather than his earlier contributions to science, that earned him a knighthood from Queen Anne in 1705.





*Newton's grave in Westminster Abbey*

Newton was made President of the Royal Society in 1703 and an associate of the French Académie des Sciences. In his position at the Royal Society, Newton made an enemy of John Flamsteed, the Astronomer Royal, by prematurely publishing Flamsteed's star catalogue, which Newton had used in his studies.

Newton died in London on March 31, 1727 [OS: March 20, 1727]<sup>[1]</sup>, and was buried in Westminster Abbey. His half-niece, Catherine Barton Conduitt,<sup>[12]</sup> served as his hostess in social affairs at his house on Jernyn Street in London; he was her "very loving Uncle,"<sup>[13]</sup> according to his letter to her when she was recovering from smallpox. Although Newton, who had no children, had divested much of his estate onto relatives in his last years he actually died intestate.

After his death, Newton's body was discovered to have had massive amounts of mercury in it, probably resulting from his alchemical pursuits. Mercury poisoning could explain Newton's eccentricity in late life.<sup>[14]</sup>

## 2 Religious views

Although the laws of motion and universal gravitation became Newton's best-known discoveries, he warned against using them to view the universe as a mere machine, as if akin to a great clock. He said, "Gravity explains the motions of the planets, but it cannot explain who set the planets in motion. God governs all things and knows all that is or can be done."<sup>[15]</sup>

His scientific fame notwithstanding, Newton's studies of the Bible and of the early Church Fathers were also noteworthy. Newton wrote works on textual criticism, most notably *An Historical Account of Two Notable Corruptions of Scripture*. He also placed the crucifixion of Jesus Christ at 3 April, AD 33, which agrees with one traditionally accepted date.<sup>[16]</sup> He also attempted, unsuccessfully, to find hidden messages within the Bible (See Bible code).

Newton may have rejected the church's doctrine of the Trinity. In a minority view, T.C. Pfizenmaier argues that he more likely held the Eastern Orthodox view of the Trinity rather than the Western one held by Roman Catholics, Anglicans, and most Protestants.<sup>[17]</sup> In his own day, he was also accused of being a Rosicrucian (as were many in the Royal Society and in the court of Charles II).<sup>[18]</sup>

In his own lifetime, Newton wrote more on religion than he did on natural science. He believed in a rationally immanent world, but he rejected the hylozoism implicit in Leibniz and Baruch Spinoza. Thus, the ordered and dynamically informed universe could be understood, and must be understood, by an active reason, but this universe, to be perfect and ordained, had to be regular.

## 2.1 Newton's effect on religious thought



*"Newton," by William Blake; here, Newton is depicted as a 'divine geometer'*

Newton and Robert Boyle's mechanical philosophy was promoted by rationalist pamphleteers as a viable alternative to the pantheists and enthusiasts, and was accepted hesitantly by orthodox preachers as well as dissident preachers like the latitudinarians.<sup>[19]</sup> Thus, the clarity and simplicity of science was seen as a way to combat the emotional and metaphysical superlatives of both superstitious enthusiasm and the threat of atheism,<sup>[20]</sup> and, at the same time, the second wave of English deists used Newton's discoveries to demonstrate the possibility of a "Natural Religion."

The attacks made against pre-Enlightenment "magical thinking," and the mystical elements of Christianity, were given their foundation with Boyle's mechanical conception of the universe. Newton gave Boyle's ideas their completion through mathematical proofs and, perhaps more importantly, was very successful in popularising them.<sup>[21]</sup> Newton refashioned the world governed by an interventionist God into a world crafted by a God that designs along rational and universal principles.<sup>[22]</sup> These principles were available for all people to discover, allowed man to pursue his own aims fruitfully in this life, not the next, and to perfect himself with his own rational powers.<sup>[23]</sup>

Newton saw God as the master creator whose existence could not be denied in the face of the grandeur of all creation.<sup>[24][25][26]</sup> But the unforeseen theological consequence of his conception of God, as Leibniz pointed out, was that God was now entirely removed from the world's affairs, since the need for intervention would only evidence some imperfection in God's creation, something impossible for a perfect and omnipotent creator.<sup>[27]</sup> Leibniz's theodicy cleared God from the responsibility for "*l'origine du mal*" by making God removed from participation in his creation. The understanding of the world was now brought down to the level of simple human reason, and humans, as Odo Marquard argued, became responsible for the correction and elimination of evil.<sup>[28]</sup>

On the other hand, latitudinarian and Newtonian ideas taken too far resulted in the millenarians, a religious faction dedicated to the concept of a mechanical universe, but finding in it the same enthusiasm and mysticism that the Enlightenment had fought so hard to extinguish.<sup>[29]</sup>

## 2.2 Views over end of the world

In a manuscript he wrote in 1704 in which he describes his attempts to extract scientific information from the Bible, he estimated that the world would end no earlier than 2060. In predicting this he said, "This I mention not to assert when the time of the end shall be, but to put a

stop to the rash conjectures of fanciful men who are frequently predicting the time of the end, and by doing so bring the sacred prophesies into discredit as often as their predictions fail."<sup>[30]</sup>

### **3 Newton and the counterfeiters**

As warden of the Royal Mint, Newton estimated that 20% of the coins taken in during The Great Recoinage were counterfeit. Counterfeiting was high treason, punishable by being hanged, drawn and quartered. Despite this, convictions of the most flagrant criminals could be extremely difficult to achieve; however, Newton proved to be equal to the task.

He gathered much of that evidence himself, disguised, while he hung out at bars and taverns. For all the barriers placed to prosecution, and separating the branches of government, English law still had ancient and formidable customs of authority. Newton was made a justice of the peace and between June 1698 and Christmas 1699 conducted some 200 cross-examinations of witnesses, informers and suspects. Newton won his convictions and in February 1699, he had ten prisoners waiting to be executed.

Possibly Newton's greatest triumph as the king's attorney was against William Chaloner. One of Chaloner's schemes was to set up phony conspiracies of Catholics and then turn in the hapless conspirators whom he entrapped. Chaloner made himself rich enough to posture as a gentleman. Petitioning Parliament, Chaloner accused the Mint of providing tools to counterfeiters (a charge also made by others). He proposed that he be allowed to inspect the Mint's processes in order to improve them. He petitioned Parliament to adopt his plans for a coinage that could not be counterfeited, while at the same time striking false coins. Newton was outraged, and went about the work to uncover anything about Chaloner. During his studies, he found that Chaloner was engaged in counterfeiting. He immediately put Chaloner on trial, but Mr Chaloner had friends in high places, and to Newton's horror, Chaloner walked free. Newton put him on trial a second time with conclusive evidence. Chaloner was convicted of high treason and hanged, drawn and quartered on 23 March 1699 at Tyburn gallows.<sup>[31]</sup>

### **4 Enlightenment philosophers**

Enlightenment philosophers chose a short history of scientific predecessors—Galileo, Boyle, and Newton principally—as the guides and guarantors of their applications of the singular concept of Nature and Natural Law to every physical and social field of the day. In this respect, the lessons of history and the social structures built upon it could be discarded.<sup>[32]</sup>

It was Newton's conception of the universe based upon Natural and rationally understandable laws that became the seed for Enlightenment ideology. Locke and Voltaire applied concepts of Natural Law to political systems advocating intrinsic rights; the physiocrats and Adam Smith applied Natural conceptions of psychology and self-interest to economic systems and the sociologists criticised the current social order for trying to fit history into Natural models of progress. Monboddo and Samuel Clarke resisted elements of Newton's work, but eventually rationalised it to conform with their strong religious views of nature.

## 5 Newton's laws of motion

The famous three laws of motion:

1. *Newton's First Law* (also known as the Law of Inertia) states that an object at rest tends to stay at rest and that an object in uniform motion tends to stay in uniform motion unless acted upon by a net external force.
2. *Newton's Second Law* states that an applied force,  $F$ , on an object equals the time rate of change of its momentum,  $p$ . Mathematically, this is written as
$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = \vec{v} \frac{dm}{dt} + m \frac{d\vec{v}}{dt}.$$
 Assuming the mass to be constant, the first term vanishes. Defining the acceleration to be  $\vec{a} = d\vec{v}/dt$  results in the famous equation  $\vec{F} = m \vec{a}$  which states that the acceleration of an object is directly proportional to the magnitude of the net force acting on the object and inversely proportional to its mass. In the MKS system of measurement, mass is given in kilograms, acceleration in metres per second squared, and force in Newtons (named in his honour).
3. *Newton's Third Law* states that for every action there is an equal and opposite reaction.

## 6 Newton's apple



A reputed descendant of Newton's apple tree, found in the Botanic Gardens in Cambridge:

When Newton saw an apple fall, he found  
In that slight startle from his contemplation —  
'Tis said (for I'll not answer above ground  
For any sage's creed or calculation) —  
A mode of proving that the earth turn'd round  
In a most natural whirl, called "gravitation;"  
And this is the sole mortal who could grapple,  
Since Adam, with a fall or with an apple.[33]

A popular story claims that Newton was inspired to formulate his theory of universal gravitation by the fall of an apple from a tree. Cartoons have gone further to suggest the apple actually hit

Newton's head, and that its impact somehow made him aware of the force of gravity. John Conduitt, Newton's assistant at the Royal Mint and husband of Newton's niece, described the event when he wrote about Newton's life:

In the year 1666 he retired again from Cambridge to his mother in Lincolnshire. Whilst he was pensively meandering in a garden it came into his thought that the power of gravity (which brought an apple from a tree to the ground) was not limited to a certain distance from earth, but that this power must extend much further than was usually thought. Why not as high as the Moon said he to himself & if so, that must influence her motion & perhaps retain her in her orbit, whereupon he fell a calculating what would be the effect of that supposition.<sup>[34]</sup>

The question was not whether gravity existed, but whether it extended so far from Earth that it could also be the force holding the moon to its orbit. Newton showed that if the force decreased as the inverse square of the distance, one could indeed calculate the Moon's orbital period, and get good agreement. He guessed the same force was responsible for other orbital motions, and hence named it "universal gravitation".

A contemporary writer, William Stukeley, recorded in his *Memoirs of Sir Isaac Newton's Life* a conversation with Newton in Kensington on 15 April 1726, in which Newton recalled "when formerly, the notion of gravitation came into his mind. It was occasioned by the fall of an apple, as he sat in contemplative mood. Why should that apple always descend perpendicularly to the ground, thought he to himself. Why should it not go sideways or upwards, but constantly to the earth's centre." In similar terms, Voltaire wrote in his *Essay on Epic Poetry* (1727), "Sir Isaac Newton walking in his gardens, had the first thought of his system of gravitation, upon seeing an apple falling from a tree." These accounts are probably exaggerations of Newton's own tale about sitting by a window in his home (Woolsthorpe Manor) and watching an apple fall from a tree.

Various trees are claimed to be "the" apple tree which Newton describes. The King's School, Grantham, claims that the tree was purchased by the school, uprooted and transported to the headmaster's garden some years later, the staff of the [now] National Trust-owned Woolsthorpe Manor dispute this, and claim that a tree present in their gardens is the one described by Newton. A descendant of the original tree can be seen growing outside the main gate of Trinity College, Cambridge, below the room Newton lived in when he studied there. The National Fruit Collection at Brogdale<sup>[35]</sup> can supply grafts from their tree (ref 1948-729), which appears identical to Flower of Kent, a coarse-fleshed cooking variety.

## 7 Writings by Newton

- *Method of Fluxions* (1671)
- *Of Natures Obvious Laws & Processes in Vegetation* (1671–75) unpublished work on alchemy<sup>[36]</sup>
- *De Motu Corporum in Gyrum* (1684)
- *Philosophiae Naturalis Principia Mathematica* (1687)
- *Opticks* (1704)
- *Reports as Master of the Mint* (1701–25)
- *Arithmetica Universalis* (1707)
- *Short Chronicle, The System of the World, Optical Lectures, The Chronology of Ancient Kingdoms, Amended* and *De mundi systemate* were published posthumously in 1728.
- *An Historical Account of Two Notable Corruptions of Scripture* (1754)

## 8 Fame

French mathematician Joseph-Louis Lagrange often said that Newton was the greatest genius who ever lived, and once added that he was also "the most fortunate, for we cannot find more than once a system of the world to establish."<sup>[37]</sup> English poet Alexander Pope was moved by Newton's accomplishments to write the famous epitaph:

Nature and nature's laws lay hid in night;

God said "Let Newton be" and all was light.

Newton himself was rather more modest of his own achievements, famously writing in a letter to Robert Hooke in February 1676:

If I have seen further it is by standing on the shoulders of giants

Historians generally think the above quote was an attack on Hooke (who was short and hunchbacked), rather than - or in addition to - a statement of modesty. The two were in a dispute over optical discoveries at the time. The latter interpretation also fits with many of his other disputes over his discoveries - such as the question of who discovered calculus as discussed above.

And then in a memoir later:

I do not know what I may appear to the world, but to myself I seem to have been only like a boy playing on the sea-shore, and diverting myself in now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me.<sup>[38]</sup>

## 9 Footnotes and references

1. <sup>^ a b c d e</sup> During Newton's lifetime, two calendars were in use in Europe: the Julian or 'Old Style' in Britain and parts of Eastern Europe, and the Gregorian or 'New Style' elsewhere. At Newton's birth, Gregorian dates were ten days ahead of Julian dates: thus Newton was born on Christmas Day, 25 December 1642 by the Julian calendar, but on 4 January 1643 by the Gregorian. Moreover, the English new year began on 25 March (the anniversary of the Incarnation) and not on 1 January (until the general adoption of the Gregorian calendar in the UK in 1753). Unless otherwise noted, the remainder of the dates in this article follow the Julian Calendar.
2. <sup>^</sup> Newton beats Einstein in polls of scientists and the public. *The Royal Society*. Retrieved on 2006-10-25.
3. <sup>^</sup> Cohen, I.B. (1970). *Dictionary of Scientific Biography*, Vol. 11, p.43. New York: Charles Scribner's Sons
4. <sup>^</sup> *Einstein and Newton 'had autism*, BBC News, 30 April 2003[1]
5. <sup>^</sup> Muir, Hazel: *Einstein and Newton showed signs of autism*, NewScientist, 30 April 2003[2]
6. <sup>^</sup> Bell, E.T. [1937] (1986). *Men of Mathematics*, Touchstone edition, New York: Simon & Schuster, pp. 91–2.
7. <sup>^</sup> Book Review Isaac Newton biography December 2003

8. ^ Biography of Isaac Newton at [www.knittingcircle.org.uk](http://www.knittingcircle.org.uk)
9. ^ Keynes, John Maynard (1972). "Newton, The Man", *The Collected Writings of John Maynard Keynes Volume X*, pp. 363–4.
10. ^ Westfall, Richard S. [1980] (1983). *Never at Rest: A Biography of Isaac Newton*. Cambridge: Cambridge University Press, pp. 530–1. notes that Newton apparently abandoned his alchemical researches.
11. ^ Dobbs, J.T. (December 1982). "Newton's Alchemy and His Theory of Matter". *Isis* **73** (4): p. 523. quoting *Opticks*
12. ^ Westfall 1980, p. 44.
13. ^ Westfall 1980, p. 595
14. ^ Newton, Isaac (1642-1727). *Eric Weisstein's World of Biography*. Retrieved on 2006-08-30.
15. ^ Tiner, J.H. (1975). *Isaac Newton: Inventor, Scientist and Teacher*. Milford, Michigan, U.S.: Mott Media.
16. ^ John P. Meier, *A Marginal Jew*, v. 1, pp. 382–402 after narrowing the years to 30 or 33, provisionally judges 30 most likely.
17. ^ Pfizenmaier, T.C. (1997). "Was Isaac Newton an Arian?". *Journal of the History of Ideas* **68** (1): pp. 57–80.
18. ^ Yates, Frances A. (1972). *The Rosicrucian Enlightenment*. London: Routledge.
19. ^ Jacob, Margaret C. (1976). *The Newtonians and the English Revolution: 1689–1720*. Cornell University Press, pp. 37,44.
20. ^ Westfall, Richard S. (1958). *Science and Religion in Seventeenth-Century England*. New Haven: Yale University Press, p. 200.
21. ^ Haakonssen, Knud. "The Enlightenment, politics and providence: some Scottish and English comparisons", in Martin Fitzpatrick ed.: *Enlightenment and Religion: Rational Dissent in eighteenth-century Britain*. Cambridge: Cambridge University Press, p. 64.
22. ^ Frankel, Charles (1948). *The Faith of Reason: The Idea of Progress in the French Enlightenment*. New York: King's Crown Press, p. 1.
23. ^ Germain, Gilbert G.. *A Discourse on Disenchantment: Reflections on Politics and Technology*, p. 28.
24. ^ Principia, Book III; cited in; Newton's Philosophy of Nature: Selections from his writings, p. 42, ed. H.S. Thayer, Hafner Library of Classics, NY, 1953.
25. ^ A Short Scheme of the True Religion, manuscript quoted in *Memoirs of the Life, Writings and Discoveries of Sir Isaac Newton* by Sir David Brewster, Edinburgh, 1850; cited in; *ibid*, p. 65.
26. ^ Webb, R.K. ed. Knud Haakonssen. "The emergence of Rational Dissent." *Enlightenment and Religion: Rational Dissent in eighteenth-century Britain*. Cambridge University Press, Cambridge: 1996. p19.
27. ^ Westfall, Richard S. *Science and Religion in Seventeenth-Century England*. p201.
28. ^ Marquard, Odo. "Burdened and Disemburdened Man and the Flight into Unindictability," in *Farewell to Matters of Principle*. Robert M. Wallace trans. London: Oxford UP, 1989.
29. ^ Jacob, Margaret C. *The Newtonians and the English Revolution: 1689–1720*. p100–101.
30. ^ *Papers Show Isaac Newton's Religious Side, Predict Date of Apocalypse*. The Associated Press (19 June 2007). Retrieved on 2007-08-01.
31. ^ Westfall 1980, pp. 571–5
32. ^ Cassels, Alan. *Ideology and International Relations in the Modern World*. p2.
33. ^ Don Juan (1821), Canto 10, Verse I. In Jerome J. McGann (ed.), *Lord Byron: The Complete Poetical Works* (1986), Vol. 5, 437
34. ^ Conduitt, John. Keynes Ms. 130.4:Conduitt's account of Newton's life at Cambridge. *Newtonproject*. Retrieved on 2006-08-30.
35. ^ [http://www.brogdale.org.uk/nfc\\_home.php](http://www.brogdale.org.uk/nfc_home.php)
36. ^ Newton's alchemical works transcribed and online at Indiana University retrieved January 11, 2007
37. ^ Fred L. Wilson, *History of Science: Newton* citing: Delambre, M. "Notice sur la vie et les ouvrages de M. le comte J. L. Lagrange," *Oeuvres de Lagrange* I. Paris, 1867, p. xx.
38. ^ *Memoirs of the Life, Writings, and Discoveries of Sir Isaac Newton* (1855) by Sir David Brewster (Volume II. Ch. 27)

## 10 Resources

### 10.1 References

- Bell, E.T. (1937). *Men of Mathematics*. New York: Simon and Schuster. ISBN 0-671-46400-0. Excerpt
- Christianson, Gale (1984). *In the Presence of the Creator: Isaac Newton & his times*. New York: Free Press. ISBN 0-02-905190-8. This well documented work provides, in particular, valuable information regarding Newton's knowledge of Patristics
- Sir Isaac Newton. *School of Mathematics and Statistics, University of St. Andrews, Scotland*. Retrieved on 8 March, 2005.
- The Newton Project. *Imperial College London*. Retrieved on 8 March, 2005.
- Westfall, Richard S. (1980, 1998). *Never at Rest*. Cambridge University Press. ISBN 0-521-27435-4.
- Craig, John (1963). "Isaac Newton and the Counterfeiters", *Notes and Records of the Royal Society (18)*. London: The Royal Society.
- "The Invisible Science." *Magical Egypt*. Chance Gardner and John Anthony West. 2005.

### 10.2 Further reading

- Berlinski, David, *Newton's Gift: How Sir Isaac Newton Unlocked the System of our World*, ISBN 0-684-84392-7 (hardback), also in paperback, Simon & Schuster, (2000).
- Christianson, Gale E. *In the Presence of the Creator: Isaac Newton and His Times*. Collier MacMillan, (1984). 608 pages.
- Dampier, William C. & M. Dampier. *Readings in the Literature of Science*. Harper & Row, New York, (1959).
- Gjertsen, Derek. *The Newton Handbook*, Routledge & Kegan Paul, (1986).
- Gleick, James. *Isaac Newton*. Knopf, (2003). hardcover, 288 pages, ISBN 0-375-42233-1.
- Hawking, Stephen, ed. *On the Shoulders of Giants*. ISBN 0-7624-1348-4 Places selections from Newton's *Principia* in the context of selected writings by Copernicus, Kepler, Galileo and Einstein.
- Hart, Michael J. *The 100*. Carol Publishing Group, (July 1992), paperback, 576 pages, ISBN 0-8065-1350-0.
- Kandaswamy, Anand M. *The Newtown/Leibniz Conflict in Context*. [3]
- Keynes, John Maynard. *Essays in Biography*. W W Norton & Co, 1963, paperback, ISBN 0-393-00189-X. Keynes had taken a close interest in Newton and owned many of Newton's private papers.
- Newton, Isaac. *Papers and Letters in Natural Philosophy*, edited by I. Bernard Cohen. Harvard University Press, 1958,1978. ISBN 0-674-46853-8.
- Newton, Isaac (1642–1727). *The Principia: a new Translation, Guide by I. Bernard Cohen* ISBN 0-520-08817-4 University of California (1999) *Warning: common mistranslations exposed!*
- Shapley, Harlow, S. Rapport, and H. Wright. *A Treasury of Science; "Newtonia"* pp. 147–9; "Discoveries" pp. 150–4. Harper & Bros., New York, (1946).
- Simmons, J. *The giant book of scientists -- The 100 greatest minds of all time*, Sydney: The Book Company, (1996).
- Richard de Villamil. *Newton, The man*. G.D. Knox, London, 1931. Preface by Albert Einstein. Reprinted by Johnson Reprint Corporation, New York (1972).



- Whiteside, D. T. *The Mathematical Papers of Isaac Newton - 8 volumes*, Cambridge University Press, Cambridge, (1967–81).
- Isaac Newton, Sir; J Edleston; Roger Cotes, *Correspondence of Sir Isaac Newton and Professor Cotes, including letters of other eminent men*, London, John W. Parker, West Strand; Cambridge, John Deighton, 1850. – Google Books
- Cohen, I. B. (1980). *The Newtonian Revolution*. Cambridge: Cambridge University Press.
- Dobbs, B. J. T. (1975). *The Foundations of Newton's Alchemy or "The Hunting of the Greene Lyon"*. Cambridge: Cambridge University Press.
- Halley, E. (1687). "Review of Newton's Principia." *Philosophical Transactions* 186:291–297.
- Herivel, J. W. (1965). *The Background to Newton's Principia. A Study of Newton's Dynamical Researches in the Years 1664–84*. Oxford: Clarendon Press.
- Koyré, A. (1965). *Newtonian Studies*. Chicago: University of Chicago Press.
- Maclaurin, C. (1748). *An Account of Sir Isaac Newton's Philosophical Discoveries, in Four Books*. London: A. Millar and J. Nourse.
- Newton, I. (1934). *Sir Isaac Newton's Mathematical Principles of Natural Philosophy and His System of the World*, tr. A. Motte, rev. F. Cajori. Berkeley: University of California Press.
- Newton, I. (1952). *Opticks, or A Treatise of the Reflections, Refractions, Inflections & Colours of Light*. New York: Dover Publications.
- Newton, I. (1958). *Isaac Newton's Papers and Letters on Natural Philosophy and Related Documents*, eds. I. B. Cohen and R. E. Schofield. Cambridge: Harvard University Press.
- Newton, I. (1959–1977). *The Correspondence of Isaac Newton*, eds. H. W. Turnbull, J. F. Scott, A. R. Hall. Cambridge: Cambridge University Press.
- Newton, I. (1962). *The Unpublished Scientific Papers of Isaac Newton: A Selection from the Portsmouth Collection in the University Library, Cambridge*, ed. A. R. Hall and M. B. Hall. Cambridge: Cambridge University Press.
- Newton, I. (1967). *The Mathematical Papers of Isaac Newton*, ed. D. T. Whiteside. Cambridge: Cambridge University Press.
- Newton, I. (1975). *Isaac Newton's 'Theory of the Moon's Motion' (1702)*. London: Dawson.
- Pemberton, H. (1728). *A View of Sir Isaac Newton's Philosophy*. London: S. Palmer.
- Stukeley, W. (1936). *Memoirs of Sir Isaac Newton's Life*, ed. A. H. White. London: Taylor and Francis.
- Westfall, R. S. (1971). *Force in Newton's Physics: The Science of Dynamics in the Seventeenth Century*. London: Macdonald.
- Shamos, Morris H. (1959). *Great Experiments in Physics*. New York: Henry Holt and Company, Inc.

## 11 See also

- De Motu (Berkeley's essay)
- Gauss-Newton algorithm
- History of calculus
- Isaac Newton's religious views
- Newton fractal
- Newton polygon
- Newton polynomial
- Newton series

- Newton v. Leibniz calculus controversy
- Newton-Cotes formulas
- Newton's cannonball
- Newton's Laws of Motion
- The Parable of the Solar System Model
- Spalding Gentlemen's Society
- "Standing on the shoulders of giants"

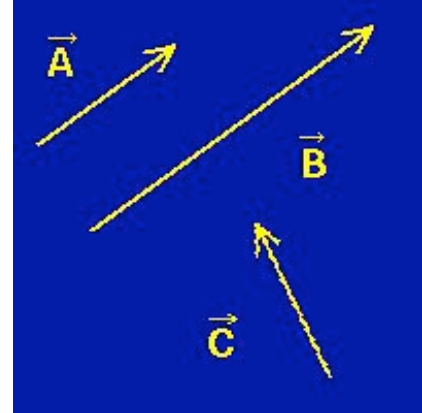
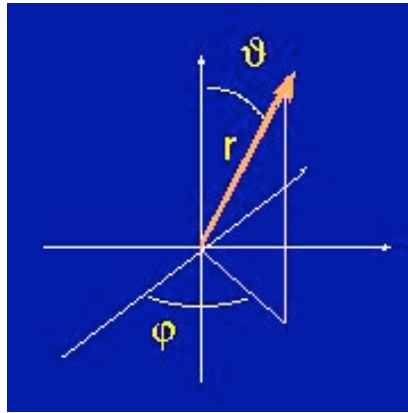
## 12 External links

- ScienceWorld biography
- *The Mind of Isaac Newton* By combining images, audio, animations and interactive segments, the application gives students a sense of Newton's multifaceted mind.
- Works by Isaac Newton at Project Gutenberg
- Newton's First ODE - A study by Phaser Scientific Software on how Newton approximated the solutions of a first-order ODE using infinite series.
- *Newton Research Project*
- PDF of Newton's Principia: 1687, 1713, and 1726 editions
- Newton's Principia - read and search
- Portraits of Isaac Newton
- Sir Isaac Newton Scientist and Mathematician
- Isaac Newton at the Open Directory Project
- Rebuttal of Newton's astrology
- Newton's Religious Views Reconsidered
- March 5–June 12, 2005 Isaac Newton's personal copy of Principia at Huntington Library
- *Newton's Royal Mint Reports*
- *Newton's Dark Secrets NOVA* TV programme.
- O'Connor, John J; Edmund F. Robertson "Isaac Newton". *MacTutor History of Mathematics archive*.
- Stanford Encyclopedia of Philosophy: Newton's views on space, time, and motion
- Newton's Castle Educational material
- The Chymistry of Isaac Newton Research on his Alchemical writings
- The Isaac Newton Institute for Mathematical Sciences
- Isaac Newton on £1 note.
- FMA Live! Cool program for teaching Newton's laws to kids
- Newton's religious position
- The "General Scholium" to Newton's Principia
- Pastore, Giovanni, *Antikythera E I Regoli Calcolatori*, Rome, 2006, privately published
- The Antikythera Calculator (Italian and English versions)

# Appendix A

## Vectors: Velocities, Accelerations, and Forces

In order to understand the discoveries of Newton, we must have an understanding of three basic quantities: (1) velocity, (2) acceleration, and (3) force. In this section we define the first two, and in the next we shall introduce forces. These three quantities have a common feature: they are what mathematicians call *vectors*.



### A1 Examples of Scalar Quantities

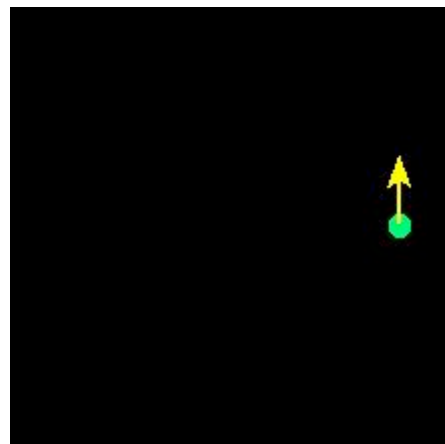
Vectors are quantities that require not only a magnitude, but a direction to specify them completely. Let us illustrate by first citing some examples of quantities that are not vectors. The number of gallons of gasoline in the fuel tank of your car is an example of a quantity that can be specified by a single number---it makes no sense to talk about a "direction" associated with the amount of gasoline in a tank. Such quantities, which can be specified by giving a single number (in appropriate units), are called *scalars*. Other examples of scalar quantities include the temperature, your weight, or the population of a country; these are scalars because they are completely defined by a single number (with appropriate units).

### A2 Examples of Vector Quantities

However, consider a velocity. If we say that a car is going 70 km/hour, we have not completely specified its motion, because we have not specified the *direction* that it is going. Thus, velocity is an example of a vector quantity. A vector generally requires more than one number to specify it; in this example we could give the magnitude of the velocity (70km/hour), a compass heading to specify the direction (say 30 degrees from North), and an number giving the vertical angle with respect to the Earth's surface (zero degrees except in chase scenes from action movies!). The adjacent figure shows a typical coordinate system for specifying a vector in terms of a length  $r$  and two angles, *theta* and *phi*.

### A3 Graphical Representation of Vectors

Vectors are often distinguished from scalar quantities either by placing a small arrow over the quantity, or by writing the quantity in a bold font. It is also common to indicate a vector by drawing an arrow whose length is proportional to the magnitude of the vector, and whose direction specifies the orientation of the vector.



In the adjacent image we show graphical representations for three vectors. Vectors A and C have the same magnitude but different directions. Vector B has the same orientation as vector A, but has a magnitude that is twice as large. Each of these represents a different vector, because for two vectors to be equivalent they must have *both* the same magnitudes and the same orientations.

#### A4 Velocity and Acceleration

Let us now give a precise definition of velocity and acceleration. They are vectors, so we must give a magnitude and a direction for them. The velocity  $v$  and the acceleration  $a$  are defined in the following illustration,

The illustration is a blue rounded rectangle. At the top, it defines velocity:  $v = \text{Velocity} = \frac{\text{Change in Position}}{\text{Change in Time}} \equiv \frac{\Delta x}{\Delta t}$ . Below that, it defines acceleration:  $a = \text{Acceleration} = \frac{\text{Change in Velocity}}{\text{Change in Time}} \equiv \frac{\Delta v}{\Delta t}$ . At the bottom, there is a diagram showing two stick figures standing on the ground. A rock is falling from a higher height on the left, with a blue arrow pointing down and the text "50 meters/sec" next to it. A second rock is falling from a lower height on the right, with a blue arrow pointing down. To the right of the diagram is a white box with a black border containing the text: "Velocity is a vector, so its direction matters! Acceleration is also a vector: it has a magnitude & direction."

This illustration also demonstrates graphically that velocity (and therefore acceleration) is a vector: the direction of the rock's velocity is certainly of critical interest to the person standing under the rock in the two illustrations!

#### A5 Uniform Circular Motion is Accelerated Motion

Notice that velocity, which is a vector, is changed if either its magnitude or its direction is changed. Thus, acceleration occurs when either the magnitude or direction of the velocity (or both) are altered.

In particular, notice from the adjacent image that circular motion (even at uniform angular velocity) implies a continual acceleration, because the *direction* of the velocity (indicated by the direction of the arrow) is continuously changing, even if its magnitude (indicated by the length of the arrow) is constant. This point, that motion on a curved path is accelerated motion, will prove crucial to our subsequent understanding of motion in gravitational fields.

#### A6 How Many Accelerators Does Your Car Have?

Be aware that in popular speech acceleration is assumed to be an increase in the magnitude of the velocity. As we have just seen, acceleration also occurs when the direction of the velocity is changed, even if the magnitude is constant; furthermore, in physics a decrease in the velocity is

just as much an acceleration as an increase. Thus, your car actually has at least 3 accelerators: (1) the foot pedal called the "accelerator", that changes the magnitude of the velocity, (2) the brake, which also changes the magnitude of the velocity, and (3) the steering wheel, which changes the direction of the velocity!

## Appendix B

### Sir Isaac Newton: The Universal Law of Gravitation

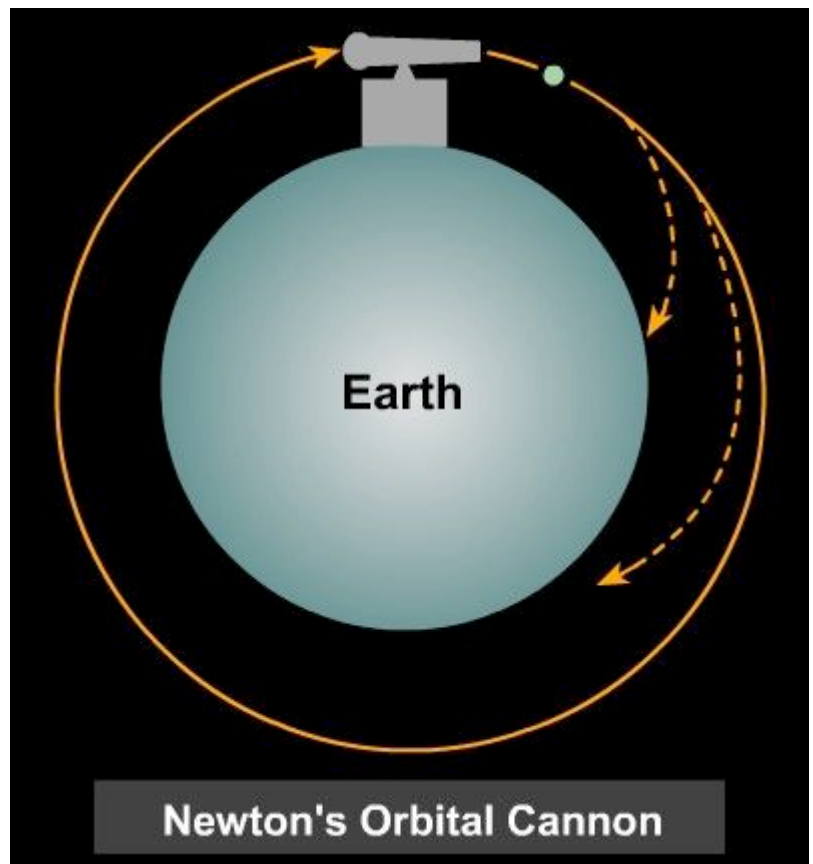
There is a popular story that Newton was sitting under an apple tree, an apple fell on his head, and he suddenly thought of the Universal Law of Gravitation. As in all such legends, this is almost certainly not true in its details, but the story contains elements of what actually happened.

#### B1 What Really Happened with the Apple?

Probably the more correct version of the story is that Newton, upon observing an apple fall from a tree, began to think along the following lines: The apple is accelerated, since its velocity changes from zero as it is hanging on the tree and moves toward the ground. Thus, by Newton's 2nd Law there must be a force that acts on the apple to cause this acceleration. Let's call this force "gravity", and the associated acceleration the "acceleration due to gravity". Then imagine the apple tree is twice as high. Again, we expect the apple to be accelerated toward the ground, so this suggests that this force that we call gravity reaches to the top of the tallest apple tree.

#### B2 Sir Isaac's Most Excellent Idea

Now came Newton's truly brilliant insight: if the force of gravity reaches to the top of the highest tree, might it not reach even further; in particular, might it not reach all the way to the orbit of the Moon! Then, the orbit of the Moon about the Earth could be a consequence of the gravitational force, because the acceleration due to gravity could change the velocity of the Moon in just such a way that it followed an orbit around the earth.




This can be illustrated with the thought experiment shown in the following figure. Suppose we fire a cannon horizontally from a high mountain; the projectile will eventually fall to earth, as indicated by the shortest trajectory in the figure, because of the gravitational force directed toward the center of the Earth and the associated acceleration. (Remember that an acceleration is a change in velocity and that velocity is a vector, so it has both a magnitude and a direction. Thus, an acceleration occurs if either or both the magnitude and the direction of the velocity change.)

But as we increase the muzzle velocity for our imaginary cannon, the projectile will travel further and further before returning to earth. Finally, Newton reasoned that if the cannon projected the cannon ball with exactly the right velocity, the projectile would travel completely around the Earth, always falling in the gravitational field but never reaching the Earth, which is curving away at the same rate that the projectile falls. That is, *the cannon ball would have been put into orbit around the Earth*. Newton concluded that the orbit of the Moon was of exactly the same nature: the Moon continuously "fell" in its path around the Earth because of the acceleration due to gravity, thus producing its orbit.

**Law of Universal Gravitation**

**Every object in the Universe attracts every other object with a force directed along the line of centers for the two objects that is proportional to the product of their masses and inversely proportional to the square of the separation between the two objects.**

$$F_g = G \frac{m_1 m_2}{r^2}$$


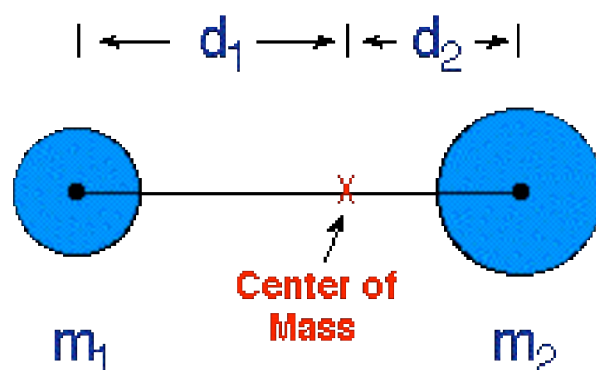
$F_g$  is the gravitational force  
 $m_1$  &  $m_2$  are the masses of the two objects  
 $r$  is the separation between the objects  
 $G$  is the universal gravitational constant

By such reasoning, Newton came to the conclusion that any two objects in the Universe exert gravitational attraction on each other, with the force having a universal form:

The constant of proportionality  $G$  is known as the *universal gravitational constant*. It is termed a "universal constant" because it is thought to be the same at all places and all times, and thus universally characterizes the intrinsic strength of the gravitational force.

### B3 The Center of Mass for a Binary System

If you think about it a moment, it may seem a little strange that in Kepler's Laws the Sun is fixed at a point in space and the planet revolves around it. Why is the Sun privileged? Kepler had rather mystical ideas about the Sun, endowing it with almost god-like qualities that justified its special place. However Newton, largely as a corollary of his 3rd Law, demonstrated that the situation actually was more symmetrical than Kepler imagined and that the Sun does



not occupy a privileged position; in the process he modified Kepler's 3rd Law.

Consider the diagram shown to the right. We may define a point called the *center of mass* between two objects through the equations

$$m_1 d_1 = m_2 d_2$$

$$d_1 + d_2 = R$$

where  $R$  is the total separation between the centers of the two objects. The center of mass is familiar to anyone who has ever played on a see-saw. The fulcrum point at which the see-saw will exactly balance two people sitting on either end is the center of mass for the two persons sitting on the see-saw.

Here is a Center of Mass Calculator that will help you make and visualize calculations concerning the center of mass. (*Caution:* this applet is written under Java 1.1, which is only supported by the most recent browsers. It should work on Windows systems under Netscape 4.06 or the most recent version of Internet Explorer 4.0, but may not yet work on Mac or Unix systems or earlier Windows browsers.)

#### **B4 Newton's Modification of Kepler's Third Law**

Because for every action there is an equal and opposite reaction, Newton realized that in the planet-Sun system the planet does not orbit around a stationary Sun. Instead, Newton proposed that both the planet and the Sun orbited around the common center of mass for the planet-Sun system. He then modified Kepler's 3rd Law to read,

$$(m_1 + m_2) P^2 = (d_1 + d_2)^3 = R^3$$

where  $P$  is the planetary orbital period and the other quantities have the meanings described above, with the Sun as one mass and the planet as the other mass. (As in the earlier discussion of Kepler's 3rd Law, this form of the equation assumes that masses are measured in solar masses, times in Earth years, and distances in astronomical units.) Notice the symmetry of this equation: since the masses are added on the left side and the distances are added on the right side, it doesn't matter whether the Sun is labeled with 1 and the planet with 2, or *vice-versa*. One obtains the same result in either case.

Now notice what happens in Newton's new equation if one of the masses (either 1 or 2; remember the symmetry) is very large compared with the other. In particular, suppose the Sun is labeled as mass 1, and its mass is much larger than the mass for any of the planets. Then the sum of the two masses is always approximately equal to the mass of the Sun, and if we take ratios of Kepler's 3rd Law for two different planets the masses cancel from the ratio and we are left with the original form of Kepler's 3rd Law:

$$\frac{P_1^2}{P_2^2} = \frac{R_1^3}{R_2^3}$$

Thus Kepler's 3rd Law is approximately valid because the Sun is much more massive than any of the planets and therefore Newton's correction is small. The data Kepler had access to were not good enough to show this small effect. However, detailed observations made after Kepler show that Newton's modified form of Kepler's 3rd Law is in better accord with the data than Kepler's original form.

### B5 Two Limiting Cases

We can gain further insight by considering the position of the center of mass in two limits. First consider the example just addressed, where one mass is much larger than the other. Then, we see that the center of mass for the system essentially coincides with the center of the massive object:

$$m_2 / m_1 \approx 0 \longrightarrow d_1 = \frac{m_2}{m_1} d_2 \approx 0$$

This is the situation in the Solar System: the Sun is so massive compared with any of the planets that the center of mass for a Sun-planet pair is always very near the center of the Sun. Thus, for all practical purposes the Sun IS almost (but not quite) motionless at the center of mass for the system, as Kepler originally thought.

However, now consider the other limiting case where the two masses are equal to each other. Then it is easy to see that the center of mass lies equidistant from the two masses and if they are gravitationally bound to each other, each mass orbits the common center of mass for the system lying midway between them:

$$m_1 = m_2 \longrightarrow d_1 = \frac{m_2}{m_1} d_2 = d_2$$

This situation occurs commonly with binary stars (two stars bound gravitationally to each other so that they revolve around their common center of mass). In many binary star systems the masses of the two stars are similar and Newton's correction to Kepler's 3rd Law is very large.

These limiting cases for the location of the center of mass are perhaps familiar from our aforementioned playground experience. If persons of equal weight are on a see-saw, the fulcrum must be placed in the middle to balance, but if one person weighs much more than the other person, the fulcrum must be placed close to the heavier person to achieve balance.

### B6 Weight and the Gravitational Force

We have seen that in the Universal Law of Gravitation the crucial quantity is mass. In popular language mass and weight are often used to mean the same thing; in reality they are related but quite different things. What we commonly call weight is really just the *gravitational force* exerted on an object of a certain mass. We can illustrate by choosing the Earth as one of the two masses in the previous illustration of the Law of Gravitation:



Thus, the weight of an object of mass  $m$  at the surface of the Earth is obtained by multiplying the mass  $m$  by the acceleration due to gravity,  $g$ , at the surface of the Earth. The acceleration due to gravity is approximately the product of the universal gravitational constant  $G$  and the mass of the Earth  $M$ , divided by the radius of the Earth,  $r$ , squared. (We assume the Earth to be spherical and neglect the radius of the object relative to the radius of the Earth in this discussion.) The measured gravitational acceleration at the Earth's surface is found to be about 980 cm/second/second.

### B7 Mass and Weight

Mass is a measure of how much material is in an object, but weight is a measure of the gravitational force exerted on that material in a gravitational field; thus, mass and weight are proportional to each other, with the acceleration due to gravity as the proportionality constant.

It follows that mass is constant for an object (actually this is not quite true, but we will save that surprise for our later discussion of the [Relativity Theory](#)), but weight depends on the location of the object. For example, if we transported the preceding object of mass  $m$  to the surface of the Moon, the gravitational acceleration would change because the radius and mass of the Moon both differ from those of the Earth. Thus, our object has mass  $m$  both on the surface of the Earth and on the surface of the Moon, but it will weigh much less on the surface of the Moon because the gravitational acceleration there is a factor of 6 less than at the surface of the Earth.

Weight =  $F_g = G \frac{Mm}{r^2} = mg$

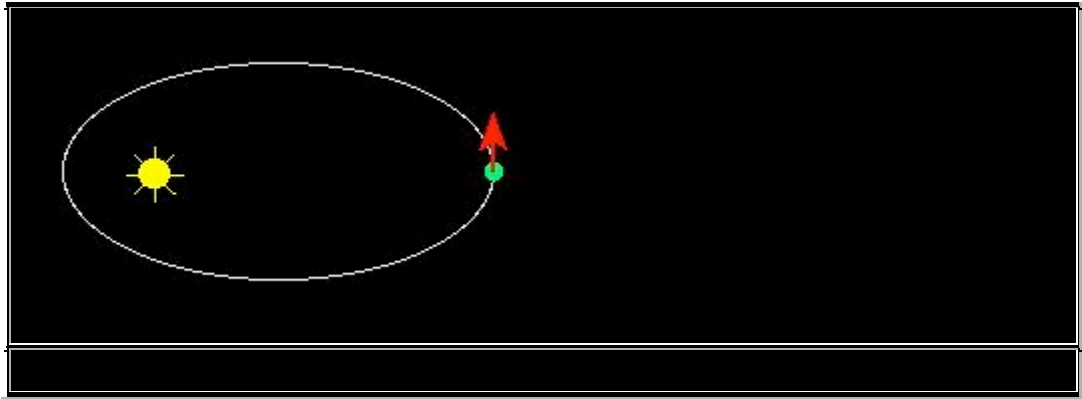
$M$  is the mass of the Earth  
 $m$  is the mass of the object  
 $r$  is the radius of the Earth  
 $g$  is the acceleration due to gravity at the Earth's surface

## Appendix C

### Newtonian Gravitation and the Laws of Kepler

We now come to the great synthesis of dynamics and astronomy accomplished by Newton: the Laws of Kepler for planetary motion may be derived from Newton's Law of Gravitation. Furthermore, Newton's Laws provide corrections to Kepler's Laws that turn out to be observable, and Newton's Law of Gravitation will be found to describe the motions of all objects in the heavens, not just the planets.

## C1 Acceleration in Keplerian Orbits



Kepler's  
Laws are  
illustrated in  
the adjacent  
animation.

The red arrow indicates the instantaneous velocity vector at each point on the orbit (as always, we greatly exaggerate the eccentricity of the ellipse for purposes of illustration). Since the velocity is a vector, the direction of the velocity vector is indicated by the direction of the arrow and the magnitude of the velocity is indicated by the length of the arrow.

Notice that (because of Kepler's 2nd Law) the velocity vector is constantly changing both its magnitude and its direction as it moves around the elliptical orbit (if the orbit were circular, the magnitude of the velocity would remain constant but the direction would change continuously). Since either a change in the magnitude or the direction of the velocity vector constitutes an acceleration, there is a continuous acceleration as the planet moves about its orbit (whether circular or elliptical), and therefore by Newton's 2nd Law there is a force that acts at every point on the orbit. Furthermore, the force is not constant in magnitude, since the change in velocity (acceleration) is larger when the planet is near the Sun on the elliptical orbit.

## C2 Newton's Laws and Kepler's Laws

Since this is a survey course, we shall not cover all the mathematics, but we now outline how Kepler's Laws are implied by those of Newton, and use Newton's Laws to supply corrections to Kepler's Laws.

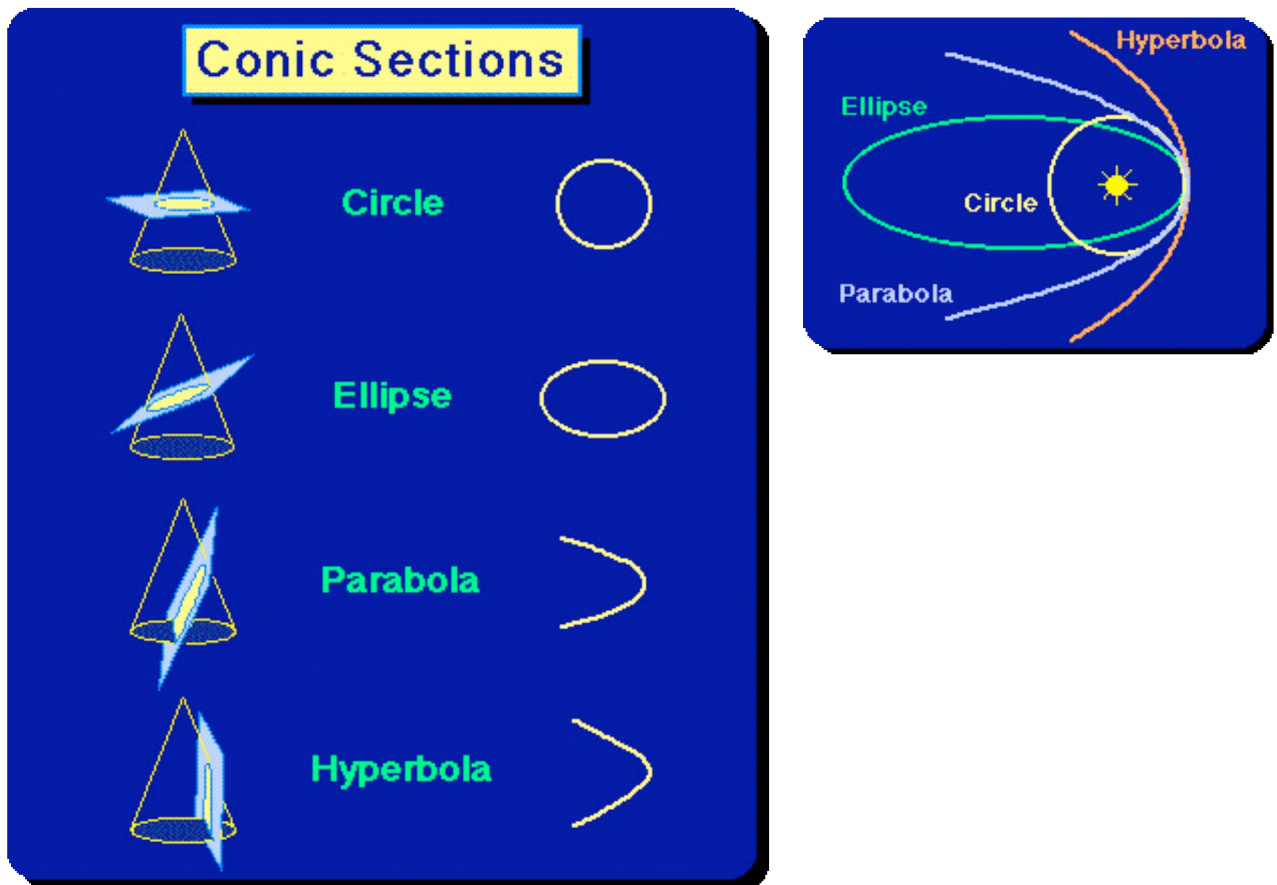
1. Since the planets move on ellipses (Kepler's 1st Law), they are continually accelerating, as we have noted above. As we have also noted above, this implies a force acting continuously on the planets.
2. Because the planet-Sun line sweeps out equal areas in equal times (Kepler's 2nd Law), it is possible to show that the force must be directed toward the Sun from the planet.
3. From Kepler's 1st Law the orbit is an ellipse with the Sun at one focus; from Newton's laws it can be shown that this means that the magnitude of the force must vary as one over the square of the distance between the planet and the Sun.
4. Kepler's 3rd Law and Newton's 3rd Law imply that the force must be proportional to the product of the masses for the planet and the Sun.

Thus, Kepler's laws and Newton's laws taken together imply that the force that holds the planets in their orbits by continuously changing the planet's velocity so that it follows an elliptical path is (1) directed toward the Sun from the planet, (2) is proportional to the product of masses for the Sun and planet, and (3) is inversely proportional to the square of the planet-Sun separation. This is precisely the form of the gravitational force, with the universal gravitational constant  $G$  as the constant of proportionality. Thus, Newton's laws of motion, with a gravitational force

used in the 2nd Law, imply Kepler's Laws, and the planets obey the same laws of motion as objects on the surface of the Earth!

### C3 Conic Sections and Gravitational Orbits

The ellipse is not the only possible orbit in a gravitational field. According to Newton's analysis, the possible orbits in a gravitational field can take the shape of the figures that are known as *conic sections* (so called because they may be obtained by slicing sections from a cone, as illustrated in the following figure).



For the ellipse (and its special case, the circle), the plane intersects opposite "edges" of the cone. For the parabola the plane is parallel to one edge of the cone; for the hyperbola the plane is not parallel to an edge but it does not intersect opposite "edges" of the cone. (Remember that these cones extend forever downward; we have shown them with bottoms because we are only displaying a portion of the cone.)